Centennial Mu Alpha Theta

April 6, 2024

Algebra Round

Do not begin until instructed to do so.

This is the Algebra Round test for the 2024 DECAGON Math Tournament. You will have 50 minutes to complete 15 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later.

Name: _____ Competitor ID: _____ Team ID: _____

1. _____	2. _____	3. _____
4. _____	5. _____	6. _____
7. _____	8. _____	9. _____
10. _____	11. _____	12. _____
13. _____	14. _____	15. _____

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1. Seol has 2 nickels, 7 dimes, and 3 quarters. What is the difference between the greatest and least amount of coins they can use to buy a $1.10 toy boat?

2. If four pens and two pencils cost $14, and eight pens and ten pencils cost $30, how much do three pens and three pencils cost, in dollars?

3. What is the smallest number that can be expressed as the sum of two unique primes in two different ways? (Rearrangements do not count as different ways)

4. Vorta is thinking of a natural number. They square it, then subtract 12, then multiply the result by 5, and the resulting number is 5 times their original number. What was their original number?

5. How many 4 digit numbers end in 35 and are divisible by 15?

6. Afmo is rowing a boat across a long river. The river is 42 miles across, and each day Afmo can travel 7 miles. However, each night the river pushes Afmo back 4 miles while they are sleeping. How many days will it take Afmo to reach the other side?

7. A roadrunner and a coyote are racing on a street. The roadrunner gives the coyote a 100 yard headstart, and catches up with the coyote after 25 seconds. If the roadrunner finishes the race 15 seconds later, how much further does the coyote have to run at the moment the roadrunner finishes, in yards? Assume both creatures run at constant speeds.

8. A set of students are playing musical chairs. They first set up their chairs in 6 identical rows, and one student is left without a chair, and eliminated. They then set up their chairs in 7 identical rows, and once more one student does not have a chair and is eliminated. They lastly set up their chairs in 8 identical rows, and one student does not have a chair. What is the smallest possible original amount of students?

9. How many positive integer divisors does $(10!)(9!)(8!)$ have?

10. Esher has a book with 362 pages, numbered from 1 to 362. They remove 24 pages (12 sheets of paper) from the middle, and now the sum of all of the page numbers is 61275. What was the largest page number they removed?

11. $\frac{x^2+4x+3}{4x^2+7x} = \frac{2x+4}{x^2+4x+4}$. Find the product of all x

12. $10^{2024} - 4^{1012}$ is divisible by 2^N. What is the largest possible integer N?

13. Let j, k, l be the distinct real roots of the equation $4x^3 + 2025x + 2024 = 0$. What is $(j+k)^3 + (k+l)^3 + (j+l)^3$?

14. For some real numbers a, b, c, the polynomial $f(x) = x^3 + ax^2 + x + 10$ has three distinct roots, and each root of $f(x)$ is also a root of the polynomial $g(x) = x^4 + x^3 + bx^2 + 50x + c$. What is g(2)?

15. Let a, b, c be integers, such that $c = (a + bi)^3 - 2027i$. What are the last two digits of c?