Centennial Mu Alpha Theta

April 6, 2024

Geometry Round

Do not begin until instructed to do so.

This is the Geometry Round test for the 2024 DECAGON Math Tournament. You will have 50 minutes to complete 15 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later. Please include all units

Name: _____ Competitor ID: _____ Team ID: _____

1. _____	2. _____	3. _____
4. _____	5. _____	6. _____
7. _____	8. _____	9. _____
10. _____	11. _____	12. _____
13. _____	14. _____	15. _____

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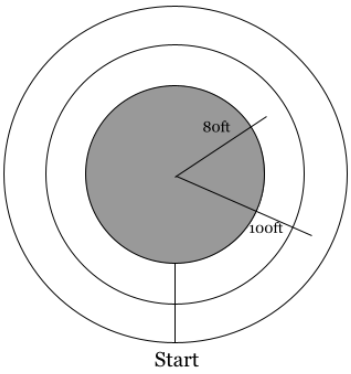
1. In a coordinate plane, points A, B, and C have coordinates $(4, 2), (7, 2)$, and $(5, 4)$ respectively. Find the area of triangle ABC.

2. Let points ABCDE be on a line in that order. The following lengths of line segments are given: BD=9, AD=12, CE=13, DE=6. Find the length of segment BE.

3. Given that the number of diagonals in a polygon can be related to the number of sides a polygon has, and that a polygon with: 3 sides has 0 diagonals, 4 sides has 2 diagonals, 5 sides has 5 diagonals, 6 sides has 9 diagonals, and 7 sides has 14 diagonals, how many diagonals will a polygon with 20 sides have?

4. A square is inscribed in a circle with a radius of $5\sqrt{2}$. What is the perimeter of the square?

5. Rajiv and Anurag are completing one lap of a circular track shown below. They start and stop on the same line segment connecting the center to a point on the outer circle, denoted "Start" on the figure below. Rajiv doesn't realize there is an advantage to starting in the first lane and chooses to race in the second lane, with radius 100 ft. If Anurag starts in the first lane with radius 80 ft and both of them stay in their lanes throughout the race, how much shorter does Anurag have to run? Express your answer in terms of π.

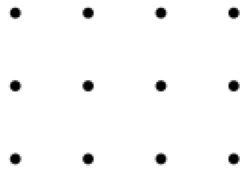


6. What is $\sin(\cos^{-1}(\frac{15}{17}))$?

7. Jaden the horse is tied on a leash to a corner of a 4-foot by 2-foot rectangular barn. What is the area Jaden can graze outside the barn if the leash is 6 feet long? Express your answer in terms of π.

8. What is the area of the polygon defined by the following coordinates: $(-2, 1), (0, 3), (3, 2), (2, -1), (-1, -2)$?

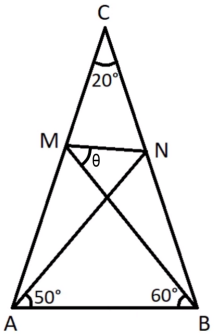
9. How many triangles can be formed with vertices at three of the 12 points in the array shown below?



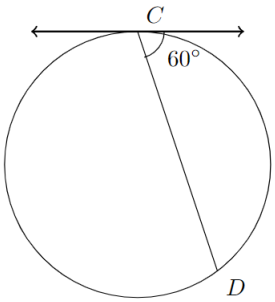
10. How many pairs of parallel edges does an icosahedron (20 sides) have?

11. Let there be a equilateral triangle WXY, with side lengths 6. Let there be a Sphere A that has diameter WX, and sphere B with diameter XY. What is the area of a circle whose circumference is formed by the intersections of the two spheres. Express your answer in terms of π.

12. Find θ in degrees in the figure below, given the angles in the figure and $\overline{AC} = \overline{BC}$.



13. A line is tangent to a unit circle at point C. A chord intersects the circle at points C and D, and the angle made by the tangent line and the chord is $60°$. The chord splits the circle into two parts. What is the difference between the larger area and the smaller area? Express your answer in simplest radical form in terms of π.



14. Martin draws a unit square. He then splits this square into four smaller squares by drawing two lines of symmetry, and shades in the top left square. He then splits the top right smaller square into four equal squares, and shades in the bottom right square. He continues this pattern of splitting and shading and rotating for eternity. What is the total area that he shades?

15. Right triangle ABC with right angle at vertex B is placed so that vertices B and C are on the circle and vertex A is outside the circle, as shown in the figure below. Line segment AB intersects the circle at point D and line segment AC intersects the circle at point E. Additionally, line segment FG is drawn so that it intersects the midpoint of segment BC, labeled by point H. If minor arc DE measures $10°$, minor arc BC measures $70°$, FH is 3 meters long, and HG is 8 meters long, find the length of AB.