

# Centennial Mu Alpha Theta

April 6, 2024

## Team Round

Do not begin until instructed to do so.

This is the Team Round test for the 2024 DECAGON Math Tournament. You will have 30 minutes to complete 10 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later.

Name: \_\_\_\_\_ Competitor ID: \_\_\_\_\_ Team ID: \_\_\_\_\_

1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

4. \_\_\_\_\_ 5. \_\_\_\_\_ 6. \_\_\_\_\_

7. \_\_\_\_\_ 8. \_\_\_\_\_ 9. \_\_\_\_\_

10. \_\_\_\_\_

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1. What is the angle difference between an internal angle of a regular square and an external angle of a regular decagon?
2. How many divisors does  $15!$  possess?
3. Albert rolls three six-sided dice, obtaining three numbers between 1 and 6, inclusive. What is the probability that the product of his rolls is composite?
4. Barry adds 4-digit number  $MATH$  to 4-digit number  $TEAM$  and gets the 5-digit number  $YUMMY$  (where distinct letters represent distinct digits between 0 and 9, inclusive). What is the value of  $M + A + T + H$ ?
5. Caleb has a triangle whose interior angles are all integers and form a geometric series with common ratio  $r$ . What is the maximum possible value of  $r$ ?
6. Daniel has a prime number  $p$  such that  $99p + 4$  is a perfect square. What is the smallest possible value of  $p$ ?
7. Evan has 10 cards numbered 1 through 10. He chooses some of the cards and takes the product of the numbers on them. When the product is divided by 3, the remainder is 1. Find the maximum number of cards he could have chosen.
8. How many polynomials of form  $P(x) = nx^2 + x + 1$  (where  $n$  is a real number) satisfy  $P(P(2)) = 0$ ?
9. Henry has an ellipse with equation  $25x^2 + 9y^2 = 225$ . What is the area of the largest triangle Henry can inscribe in this ellipse?
10. Find all values of  $x$ , real or complex, that satisfy  $(x + i)^4 + (x + 3i)^4 = 16$ .