Centennial Mu Alpha Theta

April 11, 2026

Counting and Probability Round

Do not begin until instructed to do so.

This is the Counting and Probability Round test for the 2026 DECAGON Math Tournament. You will have 45 minutes to complete 10 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later.

Name: ________________________ Competitor ID: ______ Team ID: ______

1. ______________ 2. ______________ 3. ______________

4. ______________ 5. ______________ 6. ______________

7. ______________ 8. ______________ 9. ______________

10. ______________

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1. If your name is put on a slip in a hat with the names of five other people and then thoroughly mixed, what is the probability that the slip with your name is the last one drawn if the slips are drawn without replacement?

2. A certain town uses addresses that have four digits, and each digit is 0, 1, 2 or 3. If two houses cannot have the same addresses, what is the greatest number of houses that the town can have?

3. A random integer is chosen from 10-99, inclusive. Given it is a multiple of 6, find the probability that it is a multiple of 5.

4. In a 3×3 grid, how many ways are there to place the numbers 1 to 9 so that two numbers that share an edge are not both odd or both even? Rotations and reflections are considered different.

5. Kevin is hopping randomly on the coordinate plane. Starting at $(0, 0)$, after each second he randomly moves one unit up, down, left or right with probability $\frac{1}{4}$. What is the probability that after three seconds, Kevin is within two hops of $(0, 0)$?

6. If a polygon that starts as a hexagon either gains or loses a side each second with probability 1/2, what is the likelihood that the shape ceases to be a polygon (has 2 or fewer sides) at any point in the first 6 seconds?

7. 100 points are arranged in a 10 by 10 square. If four points are chosen at random, the probability that they form a rectangle with its sides parallel to the sides of the square can be represented as $\frac{a}{b}$, where a and b share no common divisors. What is the value of a?

8. Andy, Bob, and Claire are sharing 12 identical cookies. Each cookie must be given to exactly one person.

   Claire insists on receiving at least 4 cookies, since she has a sweet tooth. Andy insists on receiving at most 3 cookies, since he finds cookies too sweet.

   In how many ways can the 12 cookies be distributed among Andy, Bob, and Claire?

9. For each integer k = 1, 2, 3, $\ldots$ 2026, let C_k be the circle $x^2 + y^2 = k$. A point is randomly chosen from the region bounded by $x^2 + y^2 \leq 2026$. What is the expected value of the number of these circles C_k that contain the point?

10. A staircase has 16 steps, numbered $0, 1, 2, \ldots, 16$. Bob starts on step 0 and wants to reach step 16.

    At each move, Bob may either go up 3 steps or go down 2 steps, as long as he stays within the staircase. Additionally, he is not allowed to visit the same step more than once during his walk.

    How many different valid sequences of moves takes Bob from step 0 to step 16?

Counting and Probability Answers

1. $\frac{1}{6}$
2. 256
3. $\frac{1}{5}$
4. 2880
5. $\frac{9}{16}$
6. $\frac{1}{8}$
7. 27
8. 30
9. $\frac{2027}{2}$
10. 24

Counting and Probability Solutions

1. The probability that your name slip is drawn last is given by multiplying the probability that your name is drawn in each successive round: $5/6 * 4/5 * 3/4 * 2/3 * 1/2 * 1/1$, which simplifies by canceling the (5*4*3*2*1) in the numerator with the (5*4*3*2*1) in the denominator, giving a probability of 1/6. Alternatively, one could observe that by symmetry, every person is equally likely to be the last person whose name is drawn, meaning that your probability is $\frac{1}{6}$

2. Suppose the number of addresses we have is x and the total number of possible addresses is T. If $x > T$ then two addresses must be the same, so $x \leq T$. Thus, we need to count the number of possible addresses. Notice that for each digit there are four choices, so there are $4^4 = \boxed{256}$ possible addresses.

3. The range $10 \cdots 99$ contains 90 consecutive integers. Since 90 is divisible by both 5 and 6, by Chinese Remainder Theorem, the conditions that a chosen number n is $n \equiv 0 \pmod 6$ and $n \equiv 0 \pmod 5$ are independent. Thus, the answer is $\boxed{\frac{1}{5}}$. Alternatively, one could find that there are 15 multiples of 6 within the range and 3 multiples of 30 and evaulate $\frac{3}{15} = \frac{1}{5}$

4. By trying out different numbers, we may see that odds must be in the corners and the center. This is because if not, then two odds will be adjacent. Then, the evens are forced into the other four squares. There are 5! ways to arrange the odds and 4! ways to arrange the even numbers in their squares, leading to an answer of $120 \times 24 = \boxed{2880}$.

5. The frog is always within two hops unless we went three hops the same direction, or two hops one direction and one hop in a perpendicular direction. There are four ways to hop three times in the same direction, and $4 \times 2 \times 3$ ways to hop one direction twice and a perpendicular once; there are 4×2 ways to choose the directions and then 3 ways to arrange which two hops are in the same direction. Thus the total number of ways to end up a distance three away is 28, so the number of ways to end up at most a distance 2 is 36. Then, the probability is $\frac{36}{64} = \boxed{\frac{9}{16}}$.

6. Since there are only six seconds and a net of four decreases must occur for the polygon to have 2 or fewer sides at the end, we can determine the maximum number of increases to be 1, except in the 1 case where 2 increases can occur in the last 2 seconds. The 1 increase can be located at any of the six seconds, and there is also the 1 possibility of no increases occuring. As such, there are 8 total possibilities, all equally likely. The possibility of any case is $\frac{1}{2}^6$, since the probability of an increase and decrease are both $\frac{1}{2}$, so the answer is $\frac{1}{2}^6 * 8 = \frac{1}{8}$.

7. There are $\binom{100}{4} = \frac{100 \times 99 \times 98 \times 97}{4 \times 3 \times 2 \times 1} = 25 \times 33 \times 49 \times 97$ ways to choose 4 points out of 100. To create a rectangle, one can choose 2 columns and 2 rows, each combination of which will create a unique rectangle. There are $\binom{10}{2} \times \binom{10}{2}$ ways to choose 2 columns and 2 rows. Therefore, $\frac{a}{b} = \frac{45 \times 45}{25 \times 33 \times 49 \times 97} = \frac{27}{11 \times 49 \times 97}$ There are no more factors of 3 in the bottom that can be canceled out with the top, giving us $a = \boxed{27}$

8. We proceed with stars and bars. Give Claire her 4 cookies in the beginning. We get $A + B + c = 8$ for non-negative solutions. $\binom{10}{2} = 45$ ways. Now count how many ways A can have more than 3. Let $A' = A - 4$, so stars and bars on $A' + B + C = 4$ gives us all cases where Andy has more than 3 cookies. This gives us 15. In total, $45 - 15 = \boxed{30}$.

9. The key observation is that a random point is equally likely to have any number of circles between 1 and 2026 that contain it. The area of each "ring" of circle k around circle k-1 is always π, since $k = r^2$ so the difference in area between C_k and $C_{k+1}$is always $\pi k - \pi(k-1) = \pi$. Therefore, the expected value is $\frac{1+2+3...2026}{2026} = \frac{2027}{2}$

10. Let a sequence of ups (u) and downs (d) represent the moves Bob makes. Taking a mod 3, we know that Bob can either take 1 or 4 downs. Bob can't have 7 downs because that would require 10 ups and there are only 16 distinct steps he can move to. We continue with casework on d. 1 down gives us 6 ups, and d cannot be placed at the beginning or at the end, thus there are 5 intervals that d can be placed in. Next, we look at 4 d's. This gives us 8 u's. However, we can't have a dudud because we would visit one step more than once. Using stars and bars and subtracting the overlapping cases, we get $\binom{7}{4} - (5 * 4 - 4) = 19$. In total, we have $\boxed{24}$ steps.