Centennial Mu Alpha Theta

April 11, 2026

Geometry Round

Do not begin until instructed to do so.

This is the Geometry Round test for the 2026 DECAGON Math Tournament. You will have 45 minutes to complete 10 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later.

Name: ________________________ Competitor ID: ______ Team ID: ______

1. ______________ 2. ______________ 3. ______________

4. ______________ 5. ______________ 6. ______________

7. ______________ 8. ______________ 9. ______________

10. ______________

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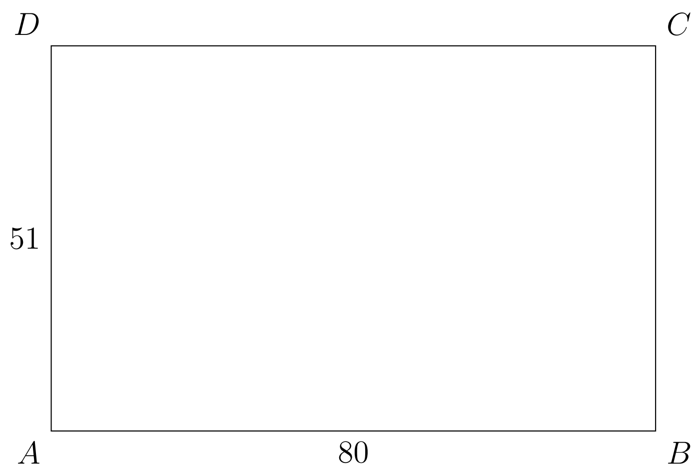
1. In triangle ABC, the midpoints of each side are labeled X, Y, and Z. What fraction of the area of triangle ABC is taken up by triangle XYZ?

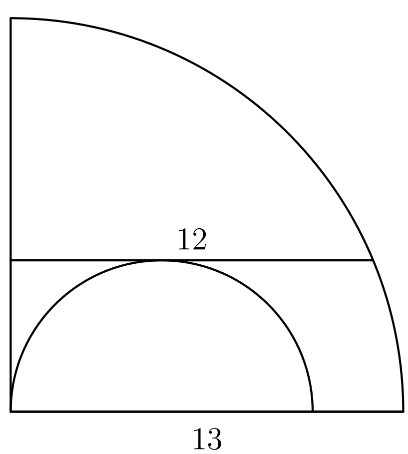
2. Nathan is standing at the origin of the coordinate plane. He throws a ball into the air and it follows the trajectory $y = -5x^2 + 10x$. As the ball is traveling, he marks the ball's starting point, highest point, and landing point. What is the area of the triangle formed by these 3 points?

3. Shamir the Laser Man is standing at corner A of a mirror room that is 51 feet wide and 80 feet long. An evil spider is resting on $\overline{CB}$ such that it is 17 feet from C and 34 feet from B. Shamir wants to shine a laser so that it bounces off $\overline{DC}$ and hits the spider immediately after, without hitting the mirrors again (lasers always bounce off flat surfaces at the same angle they hit). At what distance from D should Shamir aim the laser?



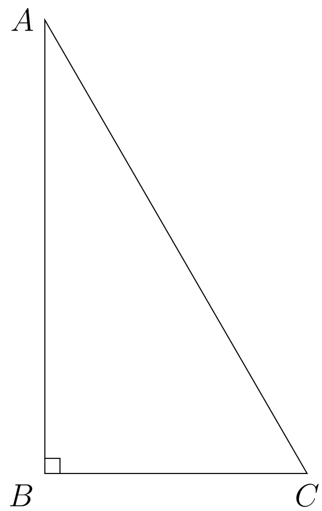
4. A quadrant (a 90-degree sector) of a circle with radius 13 is included in the diagram below. A semicircle is removed from the quadrant so that its diameter lies along the base of the quadrant, and one endpoint of the diameter is at the corner of the quadrant.

   A horizontal line segment is drawn inside the quadrant, parallel to the base. This segment is tangent to the semicircle and extends from the vertical side of the quadrant to the circular arc of the quadrant. The length of this segment is 12. Find the area of the semicircle removed from the quadrant.



5. A farmer wants to build a rectangular pen next to a barn. The barn will form one side of the rectangle, so fencing is only needed for the other three sides. If the farmer has 80 meters of fencing, what is the maximum area of the rectangular pen?

6. Right triangle $\triangle ABC$, with $\angle$B $= 90°$, $\overline{AB} = 12$, and $\overline{BC} = 4\sqrt{3}$. Point M is the midpoint of $\overline{AB}$ and point D is on $\overline{AC}$ such that $\overline{DM} = 6$. What is $\angle$BDM?



7. Justin is looking at regular polygons with an arbitrary number of sides and measures their interior angles. He observes that the interior angles of four regular polygons form an arithmetic sequence with a common difference of 30 degrees. What is the sum of the number of sides of these four regular polygons?

8. Four spheres with radius 5 are arranged so that they are all tangent to each other and stacked in a pyramid shape. What is the height of the structure?

9. Jeremy is spending a night at the museum and is trying to not be seen by a detective. The museum is comprised of five 25ft by 25ft squares shaped in a plus sign. The detective is standing in a random position inside the museum and can see Jeremy if there are no walls between each other. Jeremy doesn't know where the detective is, so he stands somewhere in the museum that minimizes the probability of being seen by the detective. What is this probability?

10. A cyclic convex quadrilateral has side lengths 15, 20, 7, and 24, in that order. Let θ be the measure of the acute angle formed by the diagonals of the quadrilateral. What is $\sin\theta$? Express your answer as a fraction in simplest form.

Geometry Answers

1. $\frac{1}{4}$
2. 5
3. 60
4. $\frac{25\pi}{2}$
5. 800
6. 60 $^\circ$
7. 25
8. $10+\frac{10\sqrt{6}}{3}$
9. $\frac{7}{10}$
10. $\frac{4}{5}$

Geometry Solutions

1. In any triangle, the triangle formed by the midpoints of the 3 sides is the same shape but half the size of the original triangle. If the side lengths are multiplied by a factor of $\frac{1}{2}$, the area is multiplied by its square, giving a proportion of $\boxed{\frac{1}{4}}$.

2. The starting and ending points can be calculated by finding the intersection of its trajectory $y = -5x^2 + 10x$ with the ground $y = 0$. Setting the two equations equal gives $5x^2 = 10x$, which has solutions $x = 0$ and $x = 2$. The maximum y-value of the trajectory intuitively occurs at the midpoint of these two points, at $x = 1$, since the trajectory is symmetric. At $x = 1$, $y = -5(1)^2 + 10(1) = 5$. This gives a triangle with base 2 and height 5, so the area is $\frac{1}{2}2(5) = \boxed{5}$.

3. By reflecting the room across $\overline{DC}$, we are able to extend the line $\overline{CB}$ 17 feet past C and draw a straight line from the reflected spider's position (which we can denote E) to A. Setting the intersection of the line $\overline{AE}$ and $\overline{DC}$ as F, we have a system of similar triangles $\triangle ADF$ and $\triangle ECF$, which solved yields $\boxed{60}$ feet from D.

4. The area of the semicircle is based on the radius of said semicircle, so we'll set this value to r. To find r, we can draw a line from the origin to the circumference so that it becomes the hypotenuse of a right triangle with r and 12. Then, we can use the Pythagorean theorem to find the length of r, being $\sqrt{169 - 144} = \sqrt{25} = 5$. We then use the formula of a semicircle, $\frac{1}{2}\pi r^2$ to find that the area of the removed semicircle is $\frac{1}{2}\pi(5)^2 = \boxed{\frac{25}{2}\pi}$

5. Let w be the width and h be the height. We have that:

$$\begin{aligned} w + 2h &= 80 \\ w &= 80 - 2h \\ A &= wh \\ A &= 80h - 2h^2 \\ A &= -2h^2 + 80h \end{aligned}$$

The maximum area occurs at the vertex of this parabola, which is:

$$h = -\frac{b}{2a} = -\frac{80}{2(-2)} = 20$$

Plugging this in, we have $A = \boxed{800 \text{ m}^2}$

6. Since we know $\overline{DM} = 6$, triangle $\triangle ABD$ must be a right triangle - $\overline{AM} = \overline{MB} = \overline{MD}$, so we can imagine a triangle inscribed in a circle with diameter $\overline{AB}$. Furthermore, the side lengths are those of a 30-60-90 triangle. Angle chasing through recognition of special and isosceles triangles yields ∠BDM $= \boxed{60°}$.

7. Notice that the least degree angles we can obtain are 60 and 90 from a triangle and a square, respectively. If the sequence began at 90, our final polygon would have to have an interior degree measure of 180, which is impossible. Thus, our interior angles have degrees of 60, 90, 120, and 150. Setting each value to the equation $\frac{180(m-2)}{n}$, which gives the degree of one of the interior angles, we find that the number of sides of each polygon must be equal to 3, 4, 6, and 12. Totally, the sum is $\boxed{25}$

8. By connecting the centers of the spheres, we discover that the height of the structure is the height of a tetrahedron plus the height of two radii. To calculate the height of the tetrahedron h, we create a right triangle with hypotenuse 10, h, and a line that extends to the center of the base. Because

the base is equilateral, we know the distance from the center is 2/3 the altitude, which is $10\sqrt{3}/3$. Calculating the height of the triangle yields $10\sqrt{2/3}$, or $\frac{10\sqrt{6}}{3}$. Adding to the 2 radii (both of length 5) results in the height of the structure being $\boxed{10 + \frac{10\sqrt{6}}{3}}$.

9. Clearly, standing in the middle square is suboptimal because the detective will always be able to see Jeremy. Intuitively, standing in the corner of one of the edge squares hides Jeremy the best, and indeed it does. It should be noted that moving towards the center is always strictly worse, while moving along the outer edge of the square does not change the area of his field of view. In this position, his field of view covers his current square, the middle square, and the opposite square, as well as half of another square, giving it an area of 3.5 big squares. Dividing this by the total 5 big squares gives a proportion of $\boxed{\frac{7}{10}}$.

10. Noticing that we know the length of all of its sides and the quadrilateral is cyclic, Ptolemy's Theorem first comes to mind. We can use it to compute the product of the diagonals (585), which can be used in the formula to find the area from the diagonals: $\text{A} = \frac{1}{2}d_1d_2\sin\theta$. Brahmagupta's Formula yields the area of 234, and we solve $\sin\theta = \boxed{\frac{4}{5}}$.