Centennial Mu Alpha Theta

April 11, 2026

Guts Round

Do not begin until instructed to do so.

Names: ______________________ Team ID: _____

1. ____________

2. ____________

3. ____________

4. ____________

1. (4 points) Find $\frac{1+2+3\cdots+2026}{1+2+3\cdots+2027}$

2. (4 points) Points A, B, C, and D are on a line. $\overline{AB}=5, \overline{BC}=4, \overline{CD}=7$. Find the difference between the maximum and minimum lengths of $\overline{AD}$

3. (4 points) What is $\frac{101^2-99^2}{101+99}$?

4. (4 points) If you have a sequence of five lights that can each be either red, green, or blue, and must be on, how many distinct color arrangements exist?

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5. ___________ 6. ___________ 7. ___________

8. ___________

5. (5 points) What is the coordinate of the vertex of the equation $x^2 = 18x - 3y + 93$?

6. (5 points) An ant walks around a ring, initially going counterclockwise. Every minute, it turns around with probability $\frac{1}{2}$, and continues otherwise. After 5 minutes, what is the expected number of minutes it has spent going clockwise?

7. (5 points) In the card game Porker, a deck consists of 52 cards, with 13 different card values from 1 to 13 and 4 different suits of hearts, clubs, spades and diamonds. A straight flush occurs when five cards are drawn such that a) all the cards are in the same suit and b) all the cards' values are consecutive (for example, $1, 2, 3, 4, 5$ and $9, 10, 11, 12, 13$ work, but $12, 13, 1, 2, 3$ does not). How many possible straight flushes can be drawn? (The order of drawing does not matter).

8. (5 points) Initially, there are 99 cakes, numbered #1, #2, #3, $\cdots$ #99. Each minute, Joe selects three distinct cakes and eats the two with the smallest and largest numbers. He continues until there is 1 cake remaining. Compute the number of different values this final cake can have.

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9. ____________ 10. ____________ 11. ____________

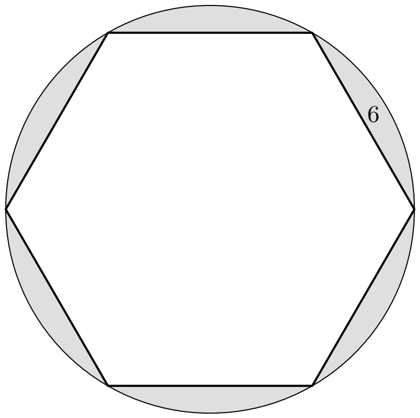
12. ____________

9. (6 points) A lattice point is a point in the coordinate plane with integer coordinates. Two different lattice points A and B are chosen with x and y coordinates between 0 and 3, inclusive. Find the number of possible lengths of segment AB.

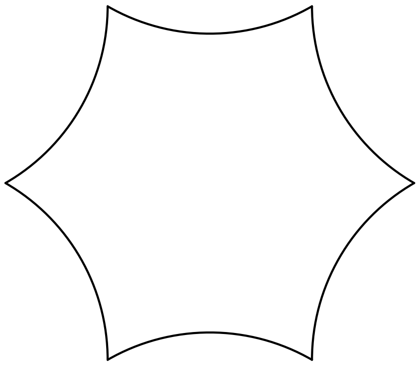
10. (6 points) In a given population, an individual can have brown, green, or blue eyes, and straight, wavy, or curly hair. There is a probability of $\frac{1}{2}$ that an individual has brown eyes, and it is twice as likely that an individual in the population has blue eyes than green eyes. Straight hair is four times more common than wavy hair, which is one twentieth as common as curly hair. If we select a person randomly, what is the chance they have blue eyes and curly hair, assuming that hair type and eye color are independent?

11. (6 points) A regular hexagon with side length 6 is inscribed into a circle. Joseph shades the area inside the circle but outside the hexagon. To form a new shape, Joseph draws a congruent hexagon and removes an area equal to the shaded area from the new hexagon to form the new shape. What is the resulting area of the new shape?

(1) Shaded: inside circle, outside hexagon



(2) New shape: Shaded segments flipped inward



12. (6 points) A graduating class has a certain number of students. When the number of students is divided by 9, the remainder is 6. When divided by 7, the remainder is 3. When divided by 5, the remainder is 2. What is the smallest number of students that can be in the graduating class?

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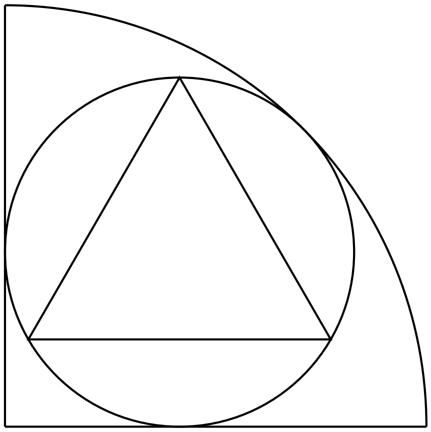
13. ____________ 14. ____________ 15. ____________

16. ____________

13. (7 points) For each positive integer n, define a_n as the sum of 2 raised to the nth power and n raised to the 2nd power. How many integers x are there such that a_{x+1} is at least double a_x?

14. (7 points) An equilateral triangle is inscribed in a circle, which is inscribed in a quadrant($\frac{1}{4}$ sector of a circle). If the area of the equilateral triangle is $\sqrt{3}$, what is the area of the quadrant?



15. (7 points) Find the last digit of $7^{(2025^{2026})}$

16. (7 points)The numbers 1-6 are placed in a 2×3 rectangular grid, one number in each cell. Compute the number of arrangements such that no two adjacent numbers have an odd product.

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17. ____________ 18. ____________ 19. ____________

20. ____________

17. (8 points) A palindrome (a number read the same forwards and backwards, e.g. 12321) is divisible by 8, 9, and 11. Find the smallest such palindrome.

18. (8 points)A triangle $\triangle ABC$ has side lengths $\overline{AB} = 8$, $\overline{BC} = 11$, and $\overline{CA} = 9$. Cevian $\overline{AD}$ splits $\overline{BC}$ into $\overline{BD} = 6$ and $\overline{DC} = 5$, and cevian $\overline{BE}$ splits $\overline{AC}$ into $\overline{AE} = 6$ and $\overline{EC} = 3$. If cevian $\overline{CF}$ intersects $\overline{AB}$ and all three cevians intersect at a singular point inside the triangle, what is the length of $\overline{AF}$?

19. (8 points) A row of 2026 boxes is arranged from left to right. The leftmost and rightmost boxes are colored red. At least one, but not all, of the remaining 2024 boxes are also colored red. The remaining boxes are to be colored so that every group of consecutive uncolored boxes is the same length, and separated by 1 red box. In how many ways can the boxes be colored under this condition?

20. (8 points) A baker can bake cakes of types A, B, C, and D. He bakes exactly one cake per day for 6 consecutive days.

    On the first day, he bakes either A or B. On the last day, he bakes either C or D. The baker is not allowed to bake the same type of cake on two consecutive days.

    How many different valid baking schedules are possible?

Guts Answers

1. $\frac{1013}{1014}$
2. 14
3. 2
4. 243
5. $(9, 58)$
6. 2
7. 36
8. 97
9. 9
10. $4/15$
11. $108\sqrt{3} - 36\pi$
12. 87
13. 2
14. $\frac{\pi}{3}(3 + 2\sqrt{2})$
15. 7
16. 72
17. 6336
18. 5
19. 13
20. 244

Guts Solutions

1. The sum can be rewritten as $\frac{\frac{2026\times 2027}{2}}{\frac{2027\times 2028}{2}} = \frac{2026}{2028} = \boxed{\frac{1013}{1014}}$

2. The maximum length of $\overline{AD}$ is if the points are in the order A,B,C,D, giving us $\overline{AD}$ $= 5 + 4 + 7 = 16$. To find the minimum length, we want to have one of the segments go "backwards". We want the amount going "forwards" and "backwards" to be as close as possible, which happens when $\overline{CD}$ goes "backwards", giving us $\overline{AD} = 5 +$ 4 - 7 = 2. 16 - 2 = $\boxed{14}$

3. Difference of squares gives $101^2 - 99^2 = (101 + 99)(101 - 99)$. Thus, the answer is $101 - 99 = \boxed{2}$.

4. Since order matter and colors can be repeated, we have 3 choices for the colors of 5 lights, so the number of possibilities is $3^5 = 243$.

5. Solve the equation for y: $y = (-x^2 + 18x + 93)/3$. Factor out the negative sign: $y = -(x^2 - 18x - 93)/3$. Create a perfect square: $y = -(x^2 - 18x + 81 - 174)/3 = -(x^2 - 18x + 81)/3 - (-174)/3 = -(x - 9)^2/3 + 58$. Recognizing this form, we can determine the vertex coordinates to be (9, 58).

6. After the first minute, the ant has a $\frac{1}{2}$ chance to be going clockwise, and for the first minute it is guaranteed to not go clockwise. Thus, the expected value is $4 \cdot 0.5 = \boxed{2}$ minutes going clockwise.

7. By the formula $A = \frac{\sqrt{3}}{4}s^2$ for an equilateral triangle, if $A = \sqrt{3}$, then $s = 2$. The height of the triangle would be $\sqrt{3}$ due to the 30-60-90 triangle ratio. As the circle is inscribed with the quadrant, the height and base of the quadrant are tangent lines to the circle, meaning that the origin of the circle is placed at the point (r,r). The distance from the origin of the quadrant, which we will call $(0, 0)$ to (r, r), is $r\sqrt{2}$. From (r, r) to the circumference, following the line from $(0, 0)$ to (r, r), the remaining distance is r. The total radius of the quadrant is then $r(\sqrt{2}+1)$. The centroid yields a ratio of $2 : 1$ for the altitude of the circle, which means that the radius of the inscribed triangle is $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$. The radius of the quadrant is therefore $\frac{2}{\sqrt{3}} * (\sqrt{2}+1)$. Using the area formula for a circle and dividing the result by four, we get that the area of the quadrant is $A_{\text{quad}} = \frac{1}{4}\pi R^2 = \frac{1}{4}\pi \left(\frac{2}{\sqrt{3}}(\sqrt{2}+1)\right)^2 = \frac{1}{4}\pi \cdot \frac{4}{3}(\sqrt{2}+1)^2 = \boxed{\frac{\pi}{3}(3 + 2\sqrt{2})}$.

8. All numbers from 2 to 98 work, for a total of $\boxed{97}$. Clearly, 1 and 99 cannot be the last cake, as they will always be eaten if there are 3 cakes left. For any other value of cake, there is at least 1 cake on both sides of it. To get cake $\#n$ as the last one, we can remove cakes until there are an equal number of cakes with lower and higher values than n, then repeatedly choose the lowest-value cake, $\#n$, and the highest-value one, eventually leaving $\#n$ as the remaining cake.

9. The length of segment AB can be calculated using the Pythagorean theorem as $\sqrt{x^2+y^2}$, where x and y are the difference in x and y coordinates. Without loss of generality, let's assume $x \geq y$ because switching x and y result in the same length. If $y=0$ or 1, then x can equal 1, 2, or 3. If $y=2$, then x can equal 2 or 3. If $y=3$, then x must equal 3. All of these combinations give a different segment length, so there are a total of $3+3+2+1=\boxed{9}$ total lengths.

10. First, we must determine the probability of selecting an individual with the specified characteristics. The probability of having brown eyes is $\frac{1}{2}$ and since blue eyes are twice as common as green eyes, the probability of having blue eyes is $\frac{1}{3}$. The probability of having curly hair can be determined by the expression 4x + x + 20x = 1. Combined like terms gives us 25x = 1, which can be solved for x = $\frac{1}{25}$. The probability of having curly hair is $\frac{20}{25}=\frac{4}{5}$. As the probability of eye color and hair type are independent, the probability of an individual with curly hair and blue eyes is $\frac{4}{5} \times \frac{1}{3} = \boxed{\dfrac{4}{15}}$

11. A regular hexagon inscribed in a circle can be partitioned into 6 congruent equilateral triangles whose side length equals the hexagon side length, 6.

    A regular hexagon can be divided into 6 congruent equilateral triangles with side length 6.

    The area of one equilateral triangle is $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(6^2) = 9\sqrt{3}$. So the area of the hexagon is $6 \cdot 9\sqrt{3} = 54\sqrt{3}$.

    Since the hexagon is inscribed in the circle, the radius of the circle is 6, so the area of the circle is $\pi(6^2) = 36\pi$.

    The shaded region inside the circle but outside the hexagon is $36\pi - 54\sqrt{3}$.

    Joseph removes this shaded area from a congruent hexagon, so the resulting area is $54\sqrt{3} - (36\pi - 54\sqrt{3}) = \boxed{108\sqrt{3} - 36\pi}$.

12. List the multiples of 9 since it is the largest of the divisors, and then add 6 to each. Then any number ending in 2 or 7 also satisfies the condition involving division by 5. For multiples of 9 up to 90, the units digit goes 9, 8, 7, 6, 5, 4, 3, 2, 1, 0. Adding 6 to each one, $36+6=42$, and $81+6=87$ are the only two possibilities for this list. We know $7 \cdot 6 = 42$, so the remainder would not be 3 but instead 0. For 87, we know $7 \cdot 12 = 84$, so the remainder is 3, which means it fits our number; thus, our answer is $\boxed{87}$. Alternatively, this problem can be solved with the Chinese Remainder Theorem.

13. a_n can be defined as $a_n = 2^n + n^2$. We want to find x such that $a_{x+1} \geq 2a_x$, or $2^{x+1} + (x+1)^2 \geq 2 * 2^x + 2x^2$, or $(x+1)^2 \geq 2x^2$. We see that only $2^2 \geq 2 * 1^2$ and $3^2 \geq 2 * 2^2$ work for $x=1$ and $x=2$, giving an answer of $\boxed{2}$.

14. By the formula $A = \frac{\sqrt{3}}{4}s^2$ for an equilateral triangle, if $A = \sqrt{3}$, then $s = 2$. The height of the triangle would be $\sqrt{3}$ due to the 30-60-90 triangle ratio. As the circle is inscribed with the quadrant, the height and base of the quadrant are tangent lines to the circle, meaning that the origin of the circle is placed at the point (r,r). The distance from the origin of the quadrant, which we will call $(0,0)$ to (r,r), is $r\sqrt{2}$. From (r,r) to the circumference, following the line from $(0,0)$ to (r,r), the remaining distance is r. The total radius of the quadrant is then $r(\sqrt{2}+1)$. The centroid yields a ratio of $2:1$ for the altitude of the circle, which means that the radius of the inscribed triangle is $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$. The radius of the quadrant is therefore $\frac{2}{\sqrt{3}} * (\sqrt{2}+1)$. Using the area formula for a circle and dividing the result by four, we get that the area of the quadrant is $A_{\text{quad}} = \frac{1}{4}\pi R^2 = \frac{1}{4}\pi\left(\frac{2}{\sqrt{3}}(\sqrt{2}+1)\right)^2 = \frac{1}{4}\pi \cdot \frac{4}{3}(\sqrt{2}+1)^2 = \boxed{\frac{\pi}{3}(3+2\sqrt{2})}$.

15. The last digits of the powers of 7 cycle between 7,9,3,1. To find the last digit, we must find what is 2025^{2026} mod 4. $2025^{2026} \mod 4 \equiv 1^{2026} \mod 4 \equiv 1 \mod 4$ Therefore, the last digit is the first one in the cycle which is 7.

16. The given condition is equivalent to having no two odd numbers being adjacent. Therefore, the odd numbers must be placed in a checkerboard pattern, of which there are 2 possible ways. Then, there are 3! ways to place the odd numbers, and 3! ways to place the even numbers. Thus, in total we have $2 \cdot 3! \cdot 3! = \boxed{72}$ arrangements.

17. Notice that $8 \times 9 \times 11 = 792$. Let us start with some kind of 4-digit number $abba$. Notice that this number is already divisible by 11 because 1001 and 110 are divisible by 11. Looking at divisibility for 9, we see that $9 \mid 2a + 2b \to 9 \mid a + b \to a + b = 9$. Now we look for a combination of a and b such that bba is divisible by 8. This means a must be even. It turns out that 6336 is divisible by 8. Therefore, the smallest such palindrome is $\boxed{6336}$.

18. By Ceva's Theorem, since $\overline{AD}, \overline{BE}, \overline{CF}$ are concurrent, $\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$. Substituting the given lengths,

$$\frac{6}{5} \cdot \frac{3}{6} \cdot \frac{AF}{FB} = 1$$

so

$$\frac{AF}{FB} = \frac{5}{3}.$$

Since

$$AF + FB = AB = 8,$$

the segment $\overline{AB}$ is divided in the ratio $5:3$. Hence

$$AF = \frac{5}{5+3} \cdot 8 = 5.$$

Therefore,

$$\boxed{5}.$$

19. There are initially 2024 uncolored boxes between the two red boxes. Suppose we divide these into n equal groups. Then there must be $n-1$ additional red boxes placed, one between each pair of groups. Thus the number of uncolored boxes remaining to be divided is 2024 - (n-1) = 2025 - n. For the spacing to be equal, this quantity must be divisible by n. Hence n must be a divisor of 2025. Since $2025 = 3^4 \cdot 5^2$, the number of positive divisors is $(4+1)(2+1) = 15$. Subtracting 2 for the cases where all 2024 middle boxes are red and where none of them are gives us $\boxed{13}$

20. Let us simplify the problem. WLOG assume the first day was A and the last day was D, but we multiply by 4 at the end. There are 3^4 possibilities in total, but we don't want D on the 5th day. That gives us 3^3 ways to have D on the 5th and 6th days, except that can't happen if D is on the fourth day. But that can't happen if D is on the third day. etc, so we have $4 \times (81 - 27 + 9 - 3 + 1) = \boxed{244}$.