Centennial Mu Alpha Theta

April 11, 2026

Team Round

Do not begin until instructed to do so.

This is the Team Round test for the 2026 DECAGON Math Tournament. You will have 45 minutes to complete 10 problems. All problems are weighted equally, but ties will be broken based on the hardest question solved (not necessarily highest numbered question). Express all answers in simplest form. Only answers recorded on the answer sheet below will be scored. Only writing tools and plain scratch paper are allowed. Assume all questions are in base 10 unless otherwise indicated. We designed this test so that most people will not be able to finish all the questions in time, so don't worry if you are struggling! Feel free to skip questions and come back to them later.

Name: ______________________   Competitor ID: ______   Team ID: ______

1. ____________   2. ____________   3. ____________

4. ____________   5. ____________   6. ____________

7. ____________   8. ____________   9. ____________

10. ____________

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1. Alice and Bob are wearing hats, knowing that each has a distinct positive integer from 2-190. They can each see the other's hat, but not their own. After both are told Alice's number is a multiple of Bob's, Alice can deduce her own number, while Bob can be sure his number is prime. Compute the minimum possible value of Bob's number.

2. Compute $9 \cdot 2^0 + 8 \cdot 2^1 + 7 \cdot 2^2 + 6 \cdot 2^3 + \cdots + 1 \cdot 2^8$.

3. Jon partially fills a rectangular prism-shaped fish tank with water to a depth of 3 ft. When the tank is laid flat, the water level is 2 ft, and when it is stood on end, the water level is 5 ft. Given that the total capacity of the tank is 810 ft^3, compute the volume of the water in the tank.

4. An ant is located in the center of a 3 by 3 grid. At any time, it can walk to another square of the grid that shares an edge. In how many ways can the ant visit every square exactly once, including the starting square?

5. In the AMC 10 competition, there are 25 questions with the following scoring:

   - 6 points for a correct answer
   - 0 points for an incorrect answer
   - 1.5 points for an unanswered question

   for a maximum of 150 points. Bob and Rob both took the exam and achieved the same total score, despite answering a different number of questions. What is the highest possible score they could have gotten?

6. Evaluate the expression

$$\sum_{n=2}^{100} \left(\frac{1}{\displaystyle\sum_{k=1}^{\infty} \frac{1}{n^k}} \right)$$

7. Call a 4-digit number a *twopair* if its digits contain 2 of the same number, and 2 of a different number. For example, 1001 and 5566 are *twopairs*, but 9999 is not. Let S be the sum of all *twopairs* less than 5000. Compute the remainder when S is divided by 11.

8. On the coordinate plane, points A and B are located at $(1, 0)$ and $(2, 0)$, respectively. Given that point C lies on the line $y = x$, what is the minimum perimeter of triangle ABC?

9. An octahedron is formed by connecting the 6 points $(1, 0, 0)$, $(-1, 0, 0)$, $(0, 1, 0)$, $(0, -1, 0)$, $(0, 0, 1)$, and $(0, 0, -1)$, creating a 3D shape with 8 faces. A cube is then formed with its vertices as the center of each face of the octahedron. Find the ratio of the volume of the octahedron to that of the cube.

10. An equilateral triangle in the plane has vertices $(x_1, 0), (x_2, 1), (x_3, 3)$. Compute $|x_1 - x_2|$.

Team Answers

1. 67
2. 1013
3. 270
4. 8
5. 132
6. 4950
7. 6
8. $\sqrt{5}+1$
9. $\frac{9}{2}$
10. $\frac{5\sqrt{3}}{3}$

Team Solutions

1. Let Alice's number be a, and Bob's be b. Since Alice can deduce her own number, there can only be 1 multiple of b less than 190, so $a = 2b$ and $3b > 190$. Bob's number is prime, and the smallest prime b such that $3b > 190$ is $\boxed{67}$.

2. Let the fraction of water in the tank be $1/x$. If the side lengths are a, b, and c, we see that 3ab = 2bc = 5ac = $\frac{abc}{x}$. Dividing by abc gives $\frac{3}{c} = \frac{2}{a} = \frac{5}{b} = \frac{1}{x}$ Then, the side lengths of the tank are $2x, 3x, 5x$, so the volume is $30x^3 = 810$ and $x = 3$. Therefore, the volume of water is $\frac{810}{3} = \boxed{270}$.

3. The ant can start by moving to any of the 4 adjacent squares. After that, it move to either of the 2 adjacent corner squares and visit the other 6 squares by continuing "clockwise" or "counterclockwise". This gives a total of $4 * 2 = \boxed{8}$ total paths.

4. If two people obtained the same score, then the difference between their numbers of unanswered questions is divisible by 4. This is because 1.5 is the only value for a question not divisible by 6, and you need $6 \div 1.5 = 4$ unanswered questions to reach the next multiple of 6. Since we want to maximize their common score, we can assume they left 0 and 4 questions blank to maximize the number of questions correct. To further maximize the score, we assume the student with 4 questions blank answered the remaining 21 questions correctly, giving a score of $21 * 6 + 4 * 1.5 = 132$. This score can also be obtained with 22 questions correct and 3 wrong, so $\boxed{132}$ is the maximum score.

5. We start with simplifying the inner sums:

$$\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \cdots = \frac{\frac{1}{n}}{1 - \frac{1}{n}} = \frac{1}{n-1}.$$

Thus the sum is

$$\sum_{n=2}^{100} \frac{1}{\frac{1}{n-1}} = \sum_{n=2}^{100} (n-1) = 1 + 2 + 3 \cdots + 99 = 4950$$

6. Twopairs of the form $\overline{aabb}$ or $\overline{abba}$ are divisible by 11, as they can be factored into $1100a + 11b$ and $1001a + 110b$, respectively. Twopairs of the form $\overline{abab}$ are equivalent to $1010a + 101b \equiv -2a + 2b \pmod{11}$. For the first term, a ranges from 1 to 4, and b can be 9 different numbers for each a, so in total a contributes $-2(1+2+3+4)(9) \equiv 7$ to the sum. Similarly, b contributes $2(45-10)(4) \equiv -1$, so the remainder is $7 - 1 = \boxed{6}$.

7. The length of AB is clearly fixed at 1, so we want to minimize the sum $AC + BC$. Reflect B over line $y = x$ to obtain point $B' = (0, 2)$. Notice that since point C lies on $y = x$, BC and $B'C$ are the same length, which reduces the problem to minimizing $AC + B'C$. However, this is just a path from point A to B'. The shortest path between two points is a straight line, so the optimal configuration is one where point C lies on segment AB'. Using the Pythagorean theorem, $AB = AC + B'C = \sqrt{2^2 + 1^2} = \sqrt{5}$. Adding the length of segment $AB = 1$, we obtain a minimum perimeter of $\boxed{\sqrt{5} + 1}$.

8. In the coordinate plane, the center of an equilateral triangle is its center of mass, or centroid. This can be calculated by taking the average the triangle's 3 vertices. Thus, the 8 vertices of the cube have coordinates $(\pm 1, \pm 1, \pm 1) \div 3 = (\pm\frac{1}{3}, \pm\frac{1}{3}, \pm\frac{1}{3})$, meaning the cube has a side legnth of $\frac{2}{3}$ and a volume of $\frac{2}{3}^3 = \frac{8}{27}$. The tetrahedron is made of 2 square pyramids with base side length $\sqrt{2}$ and height 1, so the total volume is $2 * \frac{1}{3}(\sqrt{2}^2 * 1) = \frac{4}{3}$. Finally, the ratio of the two volumes is $\frac{4}{3} \div \frac{8}{27} = \boxed{\frac{9}{2}}$.

9.